Analysis of a Low-Frequency Loudspeaker System*

PETER W. TAPPAN†

Warwick Manufacturing Corporation, 7300 N. Lehigh Avenue, Chicago 31, Illinois

A method is shown for calculating the approximate acoustic power output as a function of frequency of a low-frequency, point source, reflexed loudspeaker enclosure system employing a tube of constant cross section. The resulting power output formula is used to aid the selection of enclosure dimensions for a particular loudspeaker and amplifier. Comparison of the theoretical and measured frequency responses shows a fair correlation. Reasonably uniform response is obtained to about an octave below the free-air fundamental resonant frequency of the loudspeaker.

THE PURPOSE of this work has been to provide a theoretical analysis of the performance of the Jensen Transflex loudspeaker enclosure system, to devise from this analysis a method for selecting suitable enclosure dimensions for a given speaker and amplifier, and to verify the analysis by experiment.

THE TRANSFLEX

The Transflex[1] (Fig. 1) is described as a bass-reflex transmission-line system. Essentially, it is a modified Labyrinth[2,3] enclosure. It differs from the latter in that the absorbent lining has been omitted "to prevent loss of efficiency" and the front of the loudspeaker diaphragm has been brought within the tube mouth to achieve tight coupling between the two. This tighter coupling augments the output in the vicinity of frequencies where the frontal and tube-mouth radiation are in phase (tube length is an odd number of half wavelengths), at the expense of that at frequencies where they are nearly opposite in phase (tube length is an even number of half wavelengths). Also, the tube length is apparently made approximately a half wavelength at the free-air resonant frequency of the speaker, instead of about 0.3 wavelength. The cross-sectional area of the tube and the area of the mouth are made equal to the effective radiating area of one side of the speaker diaphragm as in the Labyrinth.

As created by Jensen, the Transflex employs a fifteen-inch driver and is designed to reproduce only the extreme low-frequency end of the range, from 45 cps on down. Above this frequency the response of the Transflex is very uneven and a crossover network is therefore used to transfer the amplifier output to another speaker. The useful range of the unit is stated to be only a little over one octave: from 45 cps, about 26 per cent above the first tube resonance, presumably to approximately 20 cps, not quite an octave below it. The theory of the Transflex is not presented in much detail and it seems probable that the design is largely the result of trial and error rather than much quantitative calculation. The writer felt that a more thorough analysis might lead to improved performance by facilitating the choice of optimum parameter values.

FREQUENCY RESPONSE OF A CLOSED-BOX SYSTEM

Because of the distributed parameters and reflex action of the Transflex, the calculation of its frequency response is rather complex. To facilitate the reader's understanding, the method to be used will therefore be illustrated first for the case of a loudspeaker in a small closed box (Fig. 2). An equivalent circuit of the impedance type for this case, with all electromagnetic quantities transformed into their

FIG. 1. Cross-sectional view of the Transflex system.

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† Senior Research Engineer

1 Jensen Mfg. Co. *Technical Bulletin No. 4,* "The Reproducer of the Future", (Dec. 1952).

2 B. J. Olney, "A Method of Eliminating Cavity Resonance, Extending Low Frequency Response, and Increasing Acoustic Damping in Cabinet-Type Loudspeakers", *J. Acoust. Soc. Am.,* 8, 104 (1936).

3 B. J. Olney, "The Acoustical Labyrinth", *Electronics,* 10, No. 4. 24 (April 1937).

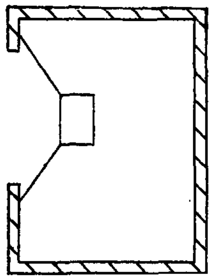

FIG. 2. Cross-sectional view of a loudspeaker in a small closed box.

acoustical equivalents, is shown in Fig. 3 (after Beranek[4]). In this circuit P_s is the instantaneous pressure‡ equivalent of the applied driving signal acting through the electromagnetic system ($P_s = e\,Bl/rS$, where e is the instantaneous open-circuit output voltage of the amplifier, B is the magnetic flux density in the gap, l is the length of voice-coil wire in the gap, r is the electrical resistance of the voice coil plus the source resistance of the amplifier, and S is the effective radiating area of one side of the diaphragm); R_e is the acoustic resistance equivalent of the electromagnetic damping ($R_e = B^2l^2/rS^2$); R_s is the acoustic resistance of the diaphragm suspension system; M_s is the effective inertance or acoustic mass of the diaphragm, voice coil, and spider; C_s is the acoustic compliance of the diaphragm suspension system; C_b is the acoustic compliance of the air confined within the box ($C_b = V/\rho c^2$, where V is the internal volume of the box, ρ is the density of air, and c is the speed of sound in air); M_b is the effective inertance of the air in contact with the back of the diaphragm (M_b varies with the geometry of the system and decreases as the box volume is reduced); M_r is the inertance of the air load on the front of the diaphragm (for radiation into a solid angle of 2π steradians, $M_r \doteq 8\rho/3\pi^2 a$ at low frequencies, where a is the effective radius of the diaphragm); and R_r is the acoustic radiation resistance of the air load on the front of the diaphragm (for radiation into 2π steradians, $R_r \doteq \rho c k^2/2\pi$ at low frequencies, where $k = 2\pi/\lambda$ and λ is the wavelength of the sound in air). This circuit is valid only at low frequencies where the wavelength is long compared to the diaphragm diameter and the longest internal dimension of the box, the diaphragm moves as a rigid piston (without "breaking up" into higher-order modes), and the inductive reactance of the voice coil is negligible compared to the dc resistance.

In that all quantities are in series, the instantaneous volume velocity U_r of air out of the system is equal to the instantaneous forward volume velocity U_s of the speaker diaphragm, and is simply the pressure equivalent P_s divided by the sum of all the impedances:

$$U_r = U_s = \frac{P_s}{R_e + R_s + j\omega M_s + 1/j\omega C_s + 1/j\omega C_b + j\omega M_b + j\omega M_r + R_r},$$

where $j = \sqrt{-1}$ and ω is the angular frequency. In a typical high-quality system R_s and R_r are usually negligible compared to R_e, so the expression may be simplified to

$$U_r = \frac{P_s}{R_e + j(\omega M_s + \omega M_b + \omega M_r - 1/\omega C_s - 1/\omega C_b)}.$$

If the rms magnitudes of P_s and U_r are p and u respectively, then the radiated acoustic power W is given by

$$W = u^2 R_r = \frac{p^2 R_r}{R_e^2 + (\omega M_s + \omega M_b + \omega M_r - 1/\omega C_s - 1/\omega C_b)^2}.$$

It would be quite tedious to calculate W as a function of frequency f from this equation. Further simplifications can, however, be made. Let f_1 be some arbitrary frequency in the range of interest. Let a number n be the ratio of frequency to f_1, i.e., $f = nf_1$. Further, let a number D_n, which we shall call the response index, be the ratio of P_s to the quantity n times U_r. Then

$$|D_n|^2 = \frac{P_s^2}{n^2 |U_r|^2}$$

$$= \frac{P_s^2}{n^2} \bigg/ \frac{P_s^2}{R_e^2 + (\omega M_s + \omega M_b + \omega M_r - 1/\omega C_s - 1/\omega C_b)^2}$$

$$= \frac{R_e^2 + (\omega M_s + \omega M_b + \omega M_r - 1/\omega C_s - 1/\omega C_b)^2}{n^2}.$$

In that R_r is proportional to f^2 and hence to n^2, it is seen that if p is independent of frequency (constant open-circuit voltage output from amplifier), then $|D_n|^2$ is inversely proportional to W. Thus, when both $|D_n|^2$ and W are expressed in db relative to their values at f_1, one is the negative of the other. Since by using $|D_n|^2$ instead of W certain factors are eliminated and the complicated term is placed

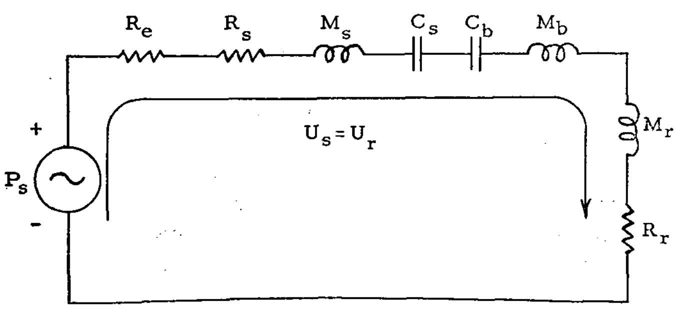

FIG. 3. Equivalent acoustical circuit of a loudspeaker in a small closed box.

[4] L. L. Beranek, *Acoustics* (McGraw-Hill Book Company, Inc., New York, 1954), p. 213.

‡ Throughout the paper "pressure" will mean pressure in excess of atmospheric, rather than absolute value.

in the numerator rather than the denominator, it is easier to calculate the relative response. If desired, $|D_n|^2$ may be further simplified by the substitution $\omega = 2\pi n f_1$ as follows:

$$|D_n|^2 = \frac{R_e^2}{n^2} + \left[2\pi f_1 (M_s + M_b + M_r) - \frac{C_b + C_s}{2\pi f_1 C_b C_s n^2} \right]^2.$$

Then, when R_e, the M's and C's are known, the relative frequency response can be immediately and readily calculated.

OUTPUT VOLUME VELOCITY OF THE TRANSFLEX

Let us return now to the Transflex. The following assumptions will be made:

1. The frequency range to be covered is low enough so that (a) the effective diameters of the speaker, tube, and port, and the distances between the mouth, port and front of the speaker diaphragm are negligible compared to a wavelength, (b) the bends in the sound path have negligible effect on the response, and (c) the speaker diaphragm moves as a rigid piston.
2. The tube walls are perfectly rigid and non-absorptive.
3. At any frequency the diaphragm displacement amplitude is proportional to the applied signal voltage.
4. The system radiates into a solid angle of 2π steradians of free space.

The symbols used will be the same as in the preceding example, except for the following additions and subtractions: Z_s is the acoustic impedance of the speaker diaphragm and voice coil in vacuum ($Z_s = R_e + jX_s$, where $X_s = \omega M_s - 1/\omega C_s$); Z_r is the acoustic radiation impedance of the external air load on the port ($Z_r = R_r + jX_r$, where $X_r = \omega M_r$ and of course R_r and M_r now load the port rather than the diaphragm directly as before); A is the internal cross-sectional area of the tube; L is the length of the tube, from the throat to the mouth or port (L is almost twice the length of the enclosure as shown in Fig. 1); $P(0)$ is the instantaneous pressure at the throat; $P(L)$ is the instantaneous pressure at the mouth, port, and front of the speaker diaphragm; U_s is again the instantaneous forward volume velocity of the speaker diaphragm (obviously, $-U_s$ equals $U(0)$, which is the instantaneous volume velocity of air into the throat); U_m is the instantaneous volume velocity of air out of the mouth; and U_r is now the instantaneous volume velocity of air out of the port (obviously, $U_r = U_s + U_m$).

An equivalent circuit of the impedance type for the Transflex with all electromagnetic quantities transformed into their acoustical equivalents, is shown in Fig. 4. The tube is shown as a delay line, with distributed inertance and compliance. In this circuit U_s is shown flowing from the throat in the direction of the generator because U_s was defined as the forward volume velocity of the speaker diaphragm (away from the throat). U_r must then flow in the same direction because $U_r = U_s + U_m$. For the same reason, U_m must flow in the same direction through Z_r as U_s. The polarity of P_s was chosen so that the generator pressure P_s creates a volume velocity U_s in the indicated direction. The polarity of $P(0)$ is as shown because a positive throat pressure $P(0)$ would tend to move the diaphragm forward, aiding the flow of U_s. The polarity of $P(L)$ is as shown because a positive mouth pressure $P(L)$ would tend to push air out of the mouth, aiding the flow of U_m.

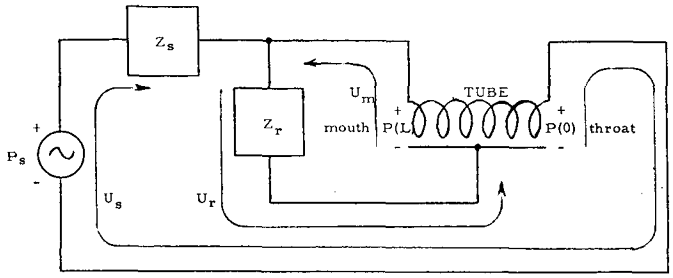

Fig. 4. Equivalent acoustical circuit of the Transflex.

From inspection of the circuit or the drawing of the enclosure, by the acoustical equivalent of Kirchoff's Second Law, two independent pressure equations can be written:

$$P(0) = -P_s + Z_s U_s + Z_r(U_s + U_m) \quad (1)$$
$$P(L) = Z_r(U_s + U_m). \quad (2)$$

What we need now are relations between the pressures and volume velocities at the ends of the tube. After Morse,[5] the pressure as a function of axial distance x along a rigid-walled, non-absorptive tube of uniform cross section, carrying axially directed sinusoidal plane waves of one frequency traveling in both directions, is of the form

$$P(x) = Q \sinh(T - jkx) \equiv Q \sinh T \cos kx - jQ \cosh T \sin kx,$$

where Q and T are complex numbers that are independent of x (but not of k) and $k = 2\pi/\lambda = \omega/c$. Similarly, the volume velocity is of the form

$$U(x) = \frac{AQ}{\rho c} \cosh(T - jkx) \equiv \frac{AQ}{\rho c} \cosh T \cos kx - j\frac{AQ}{\rho c} \sinh T \sin kx.$$

Therefore we can write

$$P(0) = Q \sinh T, \quad (3)$$
$$P(L) = Q \sinh T \cos kL - jQ \cosh T \sin kL, \quad (4)$$
$$[U(0) =] -U_s = \frac{AQ}{\rho c} \cosh T, \quad (5)$$
$$U_m = \frac{AQ}{\rho c} \cosh T \cos kL - j\frac{AQ}{\rho c} \sinh T \sin kL. \quad (6)$$

We may now combine Eqs. 1 and 3 to eliminate $P(0)$, giving

[5] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 2nd ed., 1948), p. 239.

$$-P_s + Z_s U_s + Z_r (U_s + U_m) = Q \sinh T. \quad (7)$$

Similarly, $P(L)$ may be eliminated from Eqs. 2 and 4, giving

$$Z_r (U_s + U_m) = Q \sinh T \cos kL - jQ \cosh T \sin kL. \quad (8)$$

Eqs. 5 through 8 are four simultaneous equations in the unknowns U_s, U_m, Q and T, which may be solved for $U_r = U_s + U_m$.

As a first step in the solution, the two hyperbolic functions of T may be reduced to one, at the same time eliminating Q, by rewriting Eq. 5 as

$$1 = -\frac{\rho c}{AQ \cosh T} U_s$$

and then multiplying each side of Eqs. 6, 7 and 8 by the corresponding side of this equation. We then obtain, respectively,

$$U_m = -U_s \cos kL + jU_s \tanh T \sin kL, \quad (9)$$

$$-P_s + Z_s U_s + Z_r (U_s + U_m) = -\frac{\rho c}{A} U_s \tanh T, \quad (10)$$

and

$$Z_r (U_s + U_m) = -\frac{\rho c}{A} U_s \tanh T \cos kL + j \frac{\rho c}{A} U_s \sin kL. \quad (11)$$

These three equations may be solved by straightforward algebra to give

$$U_s = \frac{[-j(A/\rho c) Z_r \sin kL - \cos kL] P_s}{\left\{ \begin{array}{c} 2Z_r (1 - \cos kL) - Z_s \cos kL \\ -j(A/\rho c) Z_s Z_r \sin kL - j(\rho c/A) \sin kL \end{array} \right\}}, \quad (12)$$

$$U_m = \frac{[1 + j(A/\rho c) Z_r \sin kL] P_s}{\left\{ \begin{array}{c} 2Z_r (1 - \cos kL) - Z_s \cos kL \\ -j(A/\rho c) Z_s Z_r \sin kL - j(\rho c/A) \sin kL \end{array} \right\}}, \quad (13)$$

and hence

$$U_r = U_s + U_m = \frac{(1 - \cos kL) P_s}{\left\{ \begin{array}{c} 2Z_r (1 - \cos kL) - Z_s \cos kL \\ -j(A/\rho c) Z_s Z_r \sin kL - j(\rho c/A) \sin kL \end{array} \right\}}. \quad (14)$$

Response Index of the Transflex

To solve for the relative frequency response, we may calculate the response index D_n as in the preceding example:

$$D_n = \frac{P_s}{n U_r} = \frac{\left\{ \begin{array}{c} 2Z_r (1 - \cos kL) - Z_s \cos kL \\ -j(A/\rho c) Z_s Z_r \sin kL - j(\rho c/A) \sin kL \end{array} \right\}}{n (1 - \cos kL)}. \quad (15)$$

Separating the real and imaginary parts, we have

$$D_n = \frac{\left\{ \begin{array}{c} [2R_r (1 - \cos kL) - R_e \cos kL + (A/\rho c)(R_r X_s \\ + R_e X_r) \sin kL] + j[2X_r (1 - \cos kL) \\ - X_s \cos kL - (A/\rho c)(R_e R_r - X_s X_r) \sin kL \\ - (\rho c/A) \sin kL] \end{array} \right\}}{n (1 - \cos kL)}. \quad (16)$$

In order to proceed, we may choose a speaker, substitute its constants into the above expression, and attempt to determine the values of the enclosure parameters for optimum performance. A medium-size, medium-price speaker was selected so that the final constants and performance of the system might be fairly typical. This speaker has a diameter of 7.5 in. and a slug magnet that appears to be about 6.8 oz of Alnico V. The pertinent constants of the speaker were measured by methods similar to those described by Beranek[6] and the results are as follows:

$R_e = 8200$ newton-sec/m^5§

$X_s = 5300\,(f/80 - 80/f)$ newton-sec/m^5.

Now for radiation into 2π steradians, the acoustic radiation resistance $R_r \doteq \rho c k^2/2\pi = .0215 f^2$ at frequencies where the wavelength is large compared to the port. At 200 cps, $R_r \doteq 860$ newton-sec/m^5. This is small compared to R_e and at lower frequencies it will of course be still smaller. The determination of the optimum values of the enclosure parameters could be greatly simplified if it could be shown that all terms containing the factor R_r in Eq. 16 are negligible. It may be seen that R_r is also small compared to the diaphragm reactance X_s at low frequencies except near the diaphragm resonance at 80 cps. For radiation into 2π steradians the acoustic radiation reactance $X_r \doteq 16\rho f/3\pi b = 2.00 f/b$, where b is the radius of the port in meters, at frequencies where the wavelength is long compared to the diameter of the port. The port area is not likely to be larger than one square foot, and will probably be somewhat smaller. Assuming an area of one square foot, the effective radius would be approximately 0.172 meter and hence X_r would be about 2300 newton-sec/m^5 at 200 cps. X_r is therefore considerably larger than R_r at this frequency and below, since R_r decreases faster with decreasing frequency than does X_r. A smaller port would obviously make X_r even larger. Thus it can be said that at and below 200 cps, R_r is small compared to R_e and X_r, and small compared to X_s except near the diaphragm resonance. Reference to Eq. 16 will then show that R_r may be neglected in the calculation of D_n except for frequencies at which the numerator of D_n becomes considerably smaller than usual, if any, since an accuracy of plus or minus one db in the value of $|D_n|^2$ will be sufficient. Essentially, the assumption we are making is that the volume velocity in the port is relatively independent of the acoustic radiation resistance. The validity of this assumption may be checked later, if desired, by Eq. 16, after the enclosure dimensions have been selected.

[6] Beranek, *op. cit.*, pp. 229-31.

§ This unit is identical to the [mks acoustical ohm].—*Ed.*

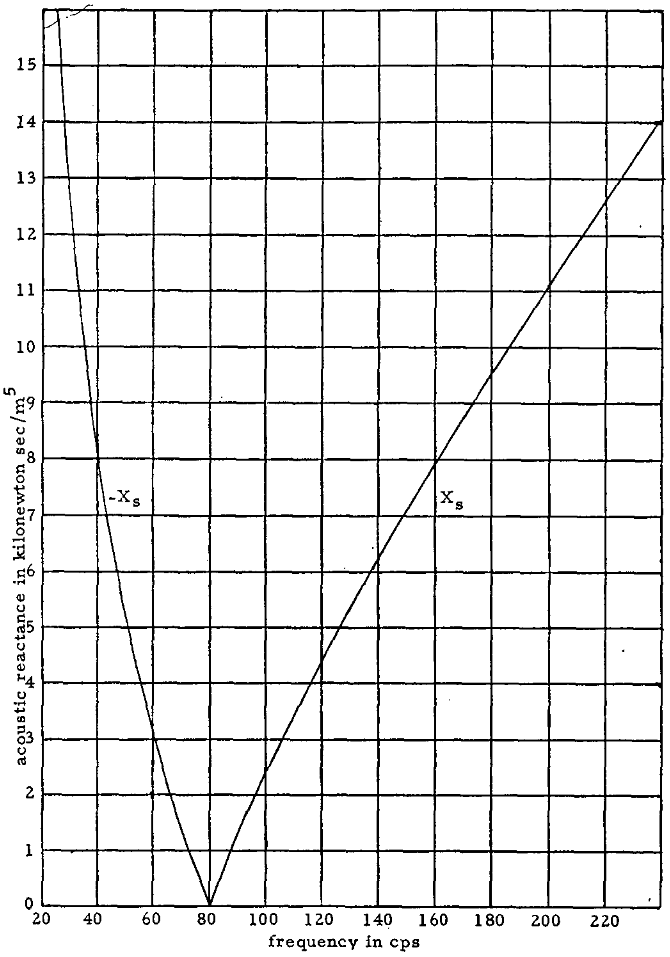

FIG. 5. Frequency dependence of the speaker acoustic reactance, X_s.

Setting $R_r = 0$ in Eq. 16, we obtain

$$D_n \doteq \frac{\left\{\begin{array}{l}[-R_e \cos kL + (A/\rho c) R_e X_r \sin kL] \\ + j\,[2X_r(1-\cos kL) - X_s \cos kL \\ + (A/\rho c) X_s X_r \sin kL - (\rho c/A) \sin kL]\end{array}\right\}}{n(1-\cos kL)}. \quad (17)$$

We may write X_r as nX_{r1}, where X_{r1} is the value of X_r at f_1. Eq. 17 may then be rewritten as

$$D_n \doteq GR_e + j\,(2X_{r1} + HJ + GX_s), \quad (18)$$

where $G = -\dfrac{\cos kL}{n(1-\cos kL)} + \left(\dfrac{\sin kL}{1-\cos kL}\right)\left(\dfrac{X_{r1}}{J}\right)$,

$H = -\dfrac{\sin kL}{n(1-\cos kL)}$,

and $J = \dfrac{\rho c}{A}$.

CHOICE OF ENCLOSURE DIMENSIONS

When the wavelength is an integral multiple of the tube length, the output will be zero because $\cos kL = 1$, making G infinite, which in turn makes $|D_n|^2$ infinite. Thus we cannot achieve flat response outside a range of kL from somewhat greater than 0 to somewhat less than 2π. It is clear that for optimally flat response over this range, the absolute value of the response index should vary as little as possible as a function of frequency. In order to see how to accomplish this, we must examine each of the parameters of the right-hand side of Eq. 18. As mentioned before, the equivalent acoustic resistance R_e of the speaker diaphragm is 8200 newton-sec/m^5, and its acoustic reactance X_s is 5300 $(f/80 - 80/f)$ newton-sec/m^5. X_s is plotted in Fig. 5. We do not yet know the values of X_{r1} and J, but they do not vary with frequency.

It will be convenient to let f_1 be the frequency at which the tube length is $\lambda/4$. At this frequency $kL = \pi/2$, so in general kL will be equal to $n\pi/2$. We may then plot H as a function of n as in Fig. 6.

Since we do not know the value of X_{r1}/J, we cannot plot G uniquely as a function of n. We may, however, plot a family of G vs n curves covering the possible range of X_{r1}/J values. The nature of G is such that for small values of X_{r1}/J, the G vs n curve is not far from the curve for $X_{r1}/J = 0$; so we may choose the latter curve as the first member of the family to be plotted, without regard to what the smallest practical value of X_{r1}/J may be. As X_{r1}/J

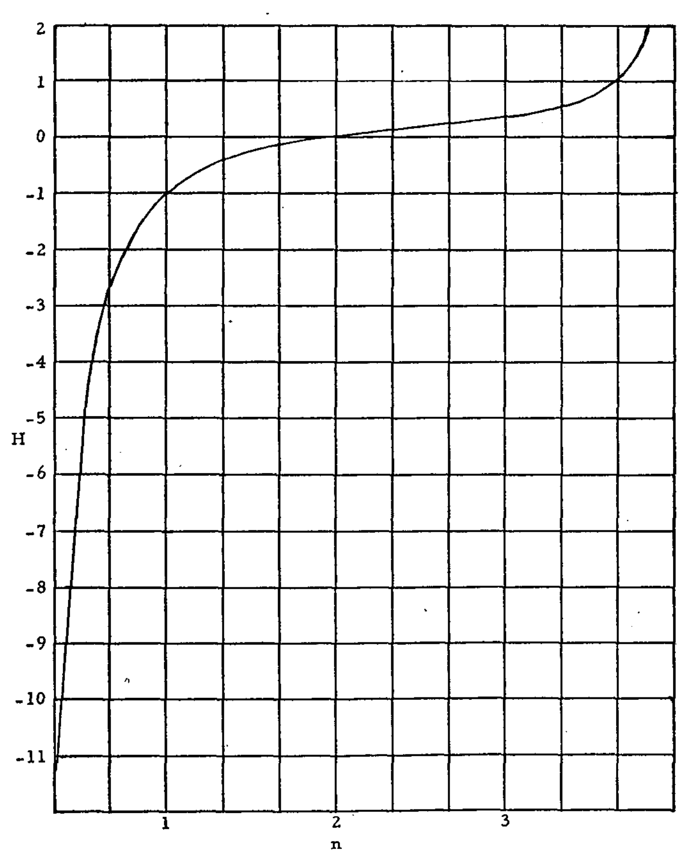

FIG. 6. H as a function of n.

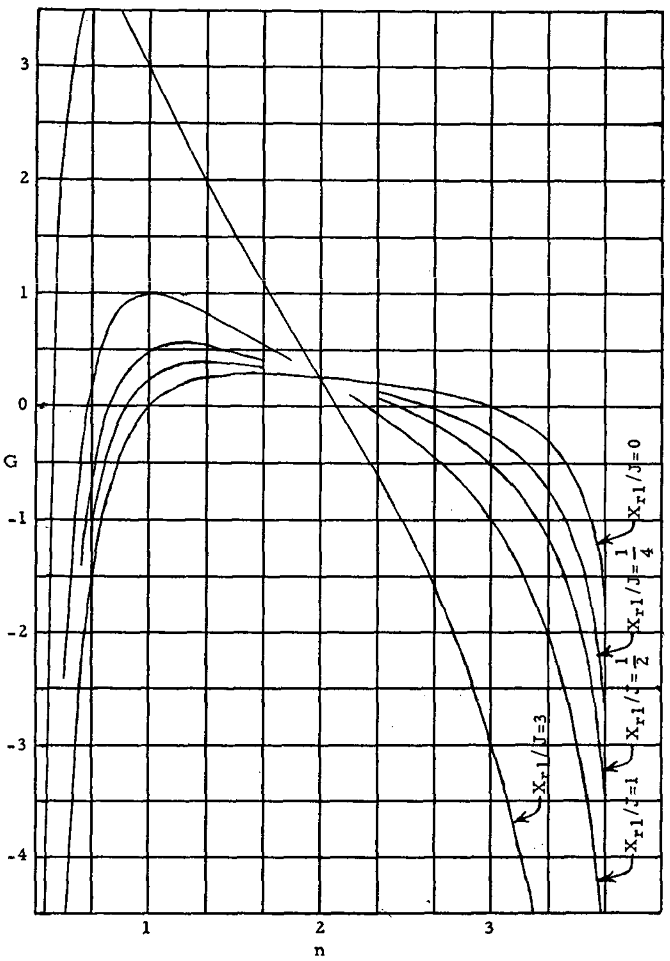

FIG. 7. G as a function of n, with X_{r1}/J as a parameter.

increases, however, the curve approaches no asymptotic limit, so we must establish an upper practical limit to X_{r1}/J. Considerations of the speaker characteristics, maximum reasonable enclosure volume, and minimum reasonable port area, which need not be elaborated here, reveal that the optimum value is most unlikely to exceed three. Fig. 7 is therefore a graph of G vs n for representative values of X_{r1}/J between zero and three.

This graph shows that G passes through zero at some value of n between one-third and one, depending on X_{r1}/J, and again between 2 and 3. By Eq. 18, when $G=0$, $|D_n|^2 = (2X_{r1}+HJ)^2$, which is independent of the speaker parameters R_e and X_s. For approximately equal response at these two frequencies, the corresponding values of $|D_n|^2$ should not differ greatly. The only frequency-dependent quantity in the above expression is H, and examination of the graph of H vs n reveals that the value of H at the lower frequency will be between -11.2 and -1.0, while that at the upper will be between zero and one-third. Therefore, the only ways in which the values of $|D_n|^2$ could be made approximately equal would be by making J small compared to $2X_{r1}$ or by making the value of D_n at the lower frequency approximately equal to the negative of that at the upper frequency. The former method is ineffective because as J/X_{r1} is decreased, the lower frequency at which $G=0$ decreases, and the resulting increase in the magnitude of the corresponding H more than offsets the decrease in J/X_{r1}. Following the latter approach, then, we pick an arbitrary value of X_{r1}/J, consult Fig. 7 to determine the n values at which $G=0$, examine Fig. 6 to learn the corresponding values of H, and then calculate and compare the resulting two values of D_n in terms of X_{r1}. The process is repeated for other values of X_{r1}/J, and it is found that values between 0.26 and 0.5 yield responses within about 2 db of each other. A value of 0.5 was initially selected for the test model.

Somewhere between the two frequencies at which $G=0$, G has a maximum, as shown in the graph. Thus, as the frequency increases, GR_e, the real part of D_n, passes through zero, increases to a maximum, and then decreases, again passing through zero. For approximately flat response over this range, then, the magnitude of the imaginary part of D_n should have a minimum at or near the frequency at which the real part has its maximum. Now, we have already made the imaginary part negative at the lower frequency at which $G=0$, and positive at the upper. Somewhere between, it must pass through zero. We shall therefore attempt to make it pass through zero at the frequency of maximum G, and make its magnitude at the frequencies at which $G=0$ approximately equal to the maximum value of the real part. When $X_{r1}/J=0.5$, the geometric mean magnitude of the imaginary part of D_n at the frequencies at which $G=0$ is approximately $2.0\,X_{r1}$. From Fig. 7, the maximum value of G occurs at about $1\frac{1}{4}\,f_1$ and is approximately 0.56. Therefore, we set $0.56\,R_e = 2.0\,X_{r1}$, obtaining $X_{r1} \doteq 2300$ newton-sec/m^5, and hence $J \doteq 4600$ newton-sec/m^5. Then, setting the imaginary part of D_n equal to zero at $1\frac{1}{4}\,f_1$, we obtain a value of about -3750 newton-sec/m^5 for X_s at that frequency; and, consulting the graph of X_s vs frequency, we learn that this value occurs at 57 cps. Therefore, $1\frac{1}{4}\,f_1 =$ 57 cps, or $f_1 = 46$ cps.

From the values of these parameters, the enclosure dimensions may be calculated. The tube length is $\lambda/4$ at f_1, so

$$L = \frac{c}{4f_1} = \frac{1131\text{ ft/sec}}{4\times 46/\text{sec}} = 6.15\text{ ft}.$$

The tube cross-sectional area A may be found from the definition of J as $\rho c/A$:

$$A = \frac{\rho c}{J} = \frac{407\text{ newton-sec/m}^3}{4600\text{ newton-sec/m}^5}$$
$$= 0.0885\text{ m}^2 = 137\text{ in.}^2 = 0.95\text{ ft}^2.$$

The volume V of air in the enclosure is thus

$$V = LA = 6.15\text{ ft}\times 0.95\text{ ft}^2 = 5.85\text{ ft}^3.$$

As discussed in the last section, the port radius b in meters

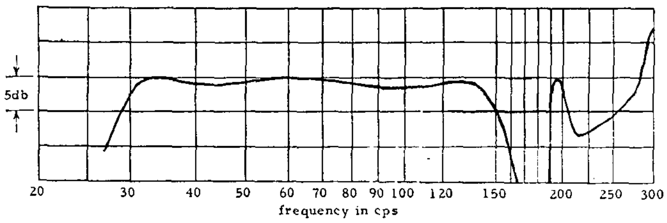

FIG. 8. Theoretical frequency response of the Transflex.

is related to the port reactance X_r in newton-sec/m^5 by the equation $X_r \doteq 2.00\, f/b$, for radiation into 2π steradians, so

$$b \doteq \frac{2.00\, f_1}{X_{r1}} = \frac{2.00 \times 46}{2300} = 0.04\,\text{m} = 1.57\,\text{in}.$$

The port area is thus $\pi(1.57\ \text{in})^2 = 7.75\ \text{in}^2$. This area will be reasonably accurate even if the port is made rectangular rather than round, provided that it is not a narrow slit.

Theoretical Performance of the Test System

Having thus tentatively selected values for the enclosure parameters, we may calculate the resulting response at numerous frequencies by Eq. 18 and plot a response curve as in Fig. 8. It is seen that the curve lies within a 5 db envelope from about 29 cps to about 150 cps, a frequency ratio of essentially five to one. The flatness and low-frequency extent of this curve are exceptionally good, considering the response usually obtained from a medium-priced speaker of this size, so the choice of enclosure parameter values may be accepted as satisfactory from the standpoint of frequency response.

As regards peak acoustic power output capability, also, the system has merit. The factor that limits the peak power output of any system is the maximum excursion capability of the diaphragm. If two systems employ different enclosures but identical speakers, and if a given diaphragm volume velocity at a given frequency produces twice the volume velocity into the external air load in one system as it does in the other, then the peak power output capability of the first system will be four times that of the other (assuming that the external dimensions of both systems are small compared to a wavelength), since the power output is proportional to the square of the volume velocity into the load.

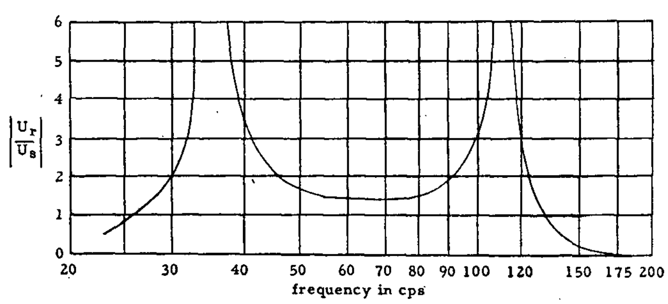

FIG. 9. Frequency dependence of $|U_r/U_s|$.

In a closed-box system, the diaphragm volume velocity *is* the volume velocity into the load. In the Transflex, U_r, the volume velocity into the load, is equal to the sum of U_s, the diaphragm volume velocity, and U_m, the volume velocity out of the mouth, and thus U_r may exceed U_s. The ratio of U_r to U_s is calculated by dividing Eq. 14 by Eq. 12, giving

$$\frac{U_r}{U_s} = \frac{1 - \cos kL}{-j\,(A/\rho c)\, Z_r \sin kL - \cos kL} = \frac{1 - \cos kL}{(X_{r1}/J)\, n \sin kL - \cos kL - j\,(R_r/J) \sin kL}.$$

It is found that the term containing R_r may be neglected except when the rest of the denominator is very small. The absolute value of U_r/U_s over the range from 23 cps to 184 cps is plotted in Fig. 9. It is seen that from 28 cps to 125 cps the absolute value of the ratio is never less than about 1.4, so that over this range the system will have at least about twice the power output capability of the same speaker in a closed box. This range covers the entire useful range of the system except for the highest frequencies of the latter, between 125 and 150 cps. The absolute value of the ratio at 150 cps is about 0.3, but this is not detrimental

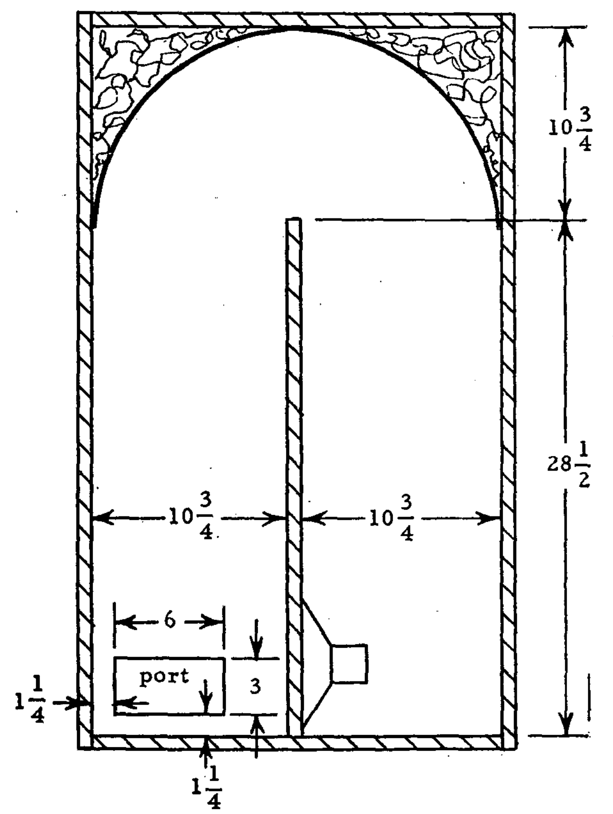

FIG. 10. Dimensions of the experimental enclosure (rear view, with rear cover removed).

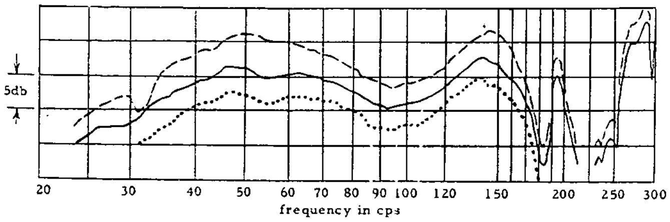

FIG. 11. Measured frequency response of the Transflex with various port areas.

because the necessary diaphragm excursion for a given power output at this frequency is still very much less than that at the lower end of the useful range.

The electroacoustical conversion efficiency of the system also may be compared to that of the same speaker in a closed box of the same volume. The power output of the closed-box system may be compared to that of the Transflex system for the same voltage input, by calculating $|D_n|^2$ as a function of frequency for the former system by the method described in the second section. It is found that the power output of the closed-box system monotonically decreases with decreasing frequency throughout the 29-150 cps range of the Transflex. At 138 cps the output is about 3 db greater than that of the Transflex, at 66 cps it is about 3 db less, and 31 cps it is approximately 16 db less.

Enclosure Construction

A drawing of the completed enclosure, with the rear panel removed to reveal the interior and the speaker in place, is shown in Fig. 10. The enclosure was constructed of ¾″ plywood. The panels were joined by glue and screws except the front panel, which was fastened with screws only, so as to be removable. The edges joining the front panel were lined with 1/16″ felt to provide an air-tight seal and prevent rattle. The 180° bend in the sound path was rounded off with a piece of ¼″ solid cardboard to help eliminate reflections. The spaces between the cardboard and the corners were filled with tightly packed paper. The port was constructed with a sliding cover to provide an area adjustable from zero to 18 in^2. The diameter of the mounting hole for the speaker was 6.5 in. The internal depth of the enclosure was 12.75 in. The other pertinent enclosure dimensions are shown in the drawing. Obviously, these dimensions were chosen to match the calculated parameter values. It should be emphasized that the experimental system was intended only as a verification of the theory, and not as a practical design suitable for commercial production. The cost of the enclosure as constructed would be out of proportion to that of the speaker used. Also, it is possible that placement of the front of the speaker outside but adjacent to the port rather than inside it, together with the use of appropriately placed absorbent lining to attenuate the tube transmission at high frequencies as in a Labyrinth, might extend the upper useful frequency limit to that inherent in the speaker.

Response Measurements

The frequency-response measurements were made in an anechoic chamber. The speaker was driven from the 4 Ω terminals of a McIntosh Model 50-W-2 power amplifier, which was in turn driven by a General Radio Model 1304-A beat-frequency oscillator with a sweep drive motor. The voltage across the speaker was set, at 100 cps, to correspond to a nominal 2.0 watts into the rated 3.2 ohms impedance of the speaker, except during some checks at other levels. The voltage was constant within a one-db envelope over the frequency range.

A General Radio Model 759 sound level meter was placed on a resilient support with its microphone 74 inches in front of the enclosure port. The absolute sound pressure level at fixed frequencies was read directly from the sound level meter; but for recording the frequency response, the sound level meter was set at its 80-db range and its output was connected to a Brüel and Kjaer Model 2301 level recorder.

Experimental Results

The frequency response of the Transflex with port openings of 4.5, 9, and 18 in^2 is shown in Fig. 11. The 9 in^2 opening corresponds fairly closely to the theoretical optimum of 7.75 in^2, especially with the added inertance of the air in the ¾″ depth of the port between the front and back surfaces of the front panel, because this depth was neglected in the calculation. The curve for the 9 in^2 port may thus be compared with the calculated response (Fig. 8). It is seen that both curves are fairly smooth between 30 and 140 cps, have a severe dip at about 184 cps, a peak at about 195 cps, and another dip at about 220 cps. However, the calculated response is flat within about a 1.5 db envelope from 32 to 140 cps, whereas the measured response requires a 7.5 db envelope over this same range, sagging somewhat at the low end and in the 90-cps region with respect to the response at other frequencies. Possible sources of error are frequency-dependent error in the measuring instruments, error in the measured constants of the speaker, incompletely valid assumptions as the basis for the theoretical calculations, imperfect absorption by the walls of the anechoic

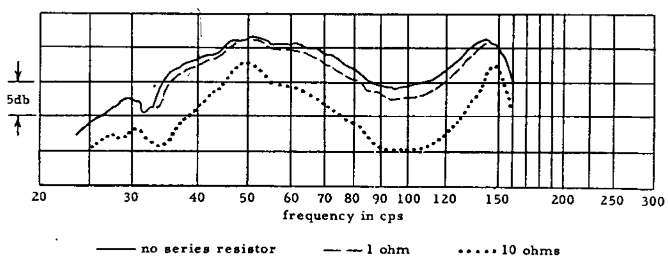

FIG. 12. Effect of series resistance upon the measured frequency response of the Transflex.

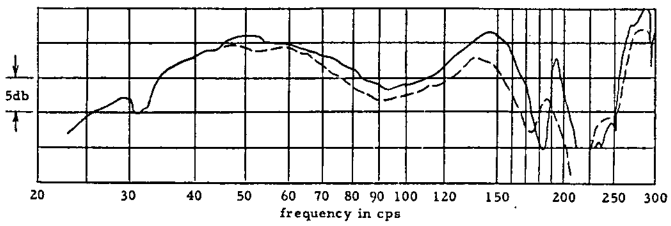

FIG. 13. Measured frequency response of the Transflex, with and without an absorbent lining.

chamber, diffraction by the external edges of the enclosure, and the fact that the effective tube length and cross-sectional area may differ from the actual values.

It can be seen that as the port area was increased, the relative frequency response changed very little but the overall efficiency increased. The change from a 4.5 in^2 port to a 9 in^2 port increased the average output by about three db, and the change from 9 in^2 to 18 in^2 raised the average level another four or five db. All subsequent measurements were made with an 18 in^2 port area. Larger port openings were not tried because there was an indication that they would have yielded a somewhat less flat frequency response.

The effect on frequency response of changing the amplifier damping factor, while maintaining constant open-circuit output voltage, is illustrated in Fig. 12. This was accomplished by placing a resistor in series with the speaker. The importance of a high damping factor (low source resistance) in maintaining the flattest possible response is apparent. It would probably be beneficial to use an amplifier with a negative source resistance to cancel part of the resistance of the voice coil.[7]

Figure 13 shows the effect on frequency response of lining portions of the lower part of the enclosure with fiberglass padding. The severity of the dip near 184 cps and the peaks near 140 and 190 cps was reduced. It is apparent that a greater amount of absorbing material would have aggravated the dip near 90 cps.

It would have been desirable to observe the effect on frequency response of varying the tube length and cross-sectional area, but unfortunately this would have necessitated virtually rebuilding the entire enclosure for each variation. It is likely that adjustment of these parameters could have yielded a more uniform response.

Since there are apparently no distortion-producing elements in the enclosure itself, the system distortion is undoubtedly generated entirely by the speaker, and therefore is of interest only in comparison to that of other enclosures using the same speaker. No comparison measurements were made, but the theoretical power output capability compared to that of the same speaker in a closed box has been discussed previously. As a rough check to insure that distortion was not affecting the response measurements, however, the output of the sound level meter was viewed on an oscilloscope, and appeared to be mostly fundamental down to 32 cps at the nominal two-watt input level.

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[7] R. E. Werner, "Effect of a Negative Impedance Source on Loudspeaker Performance", *J. Acoust. Soc. Am.*, **29**, 335 (1957). [Also *J. Audio Eng. Soc.*, **6**, 240 (1958).—*Ed.*]

THE AUTHOR

Peter W. Tappan received his B.S. in physics in 1952 and his M.S. in 1958 from the Illinois Institute of Technology. He was employed at Motorola, Inc. in Chicago for nine months during 1951. From 1951 to 1956 he worked in the Physics Department of the Armour Research Foundation, where he performed research on such varied projects as an X-ray intensification system, special tape recording heads, an electronic piano, and high-powered public address systems.

In 1956 he joined the Warwick Manufacturing Corporation as a Senior Research Engineer. He is responsible for the acoustical research of that company and has worked on the design and development of speaker systems, phonograph pickups, and stereophonic and pseudostereophonic equipment.

Mr. Tappan is a member of the Acoustical Society of America, the Chicago Acoustical and Audio Group, and the Professional Group on Audio of the Institute of Radio Engineers.